Interactive verification of Lustre programs in Vélus

Timothy Bourke **Paul Jeanmaire** Marc Pouzet

ENS, Inria Parkas team

Synchron'21, November 26

Interactive verification of synchronous programs

Goal: prove properties of streams in a program

```
node f (r : bool) returns (n : int)
var up : bool;
let
  up = true fby ((up and n < 9) or (not up and n <= 1));
  n = 0 fby (if up then n + 1 else n - 1);
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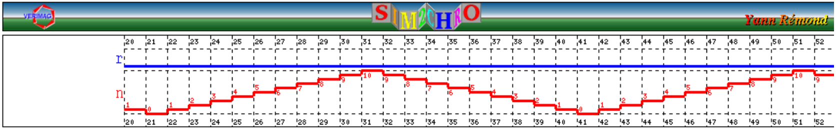

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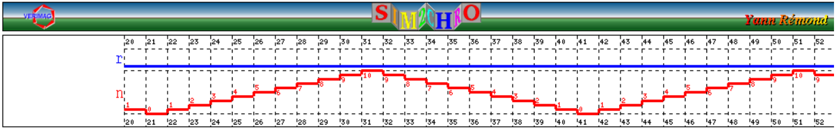
Show that values of stream `n` stay between 0 and 10



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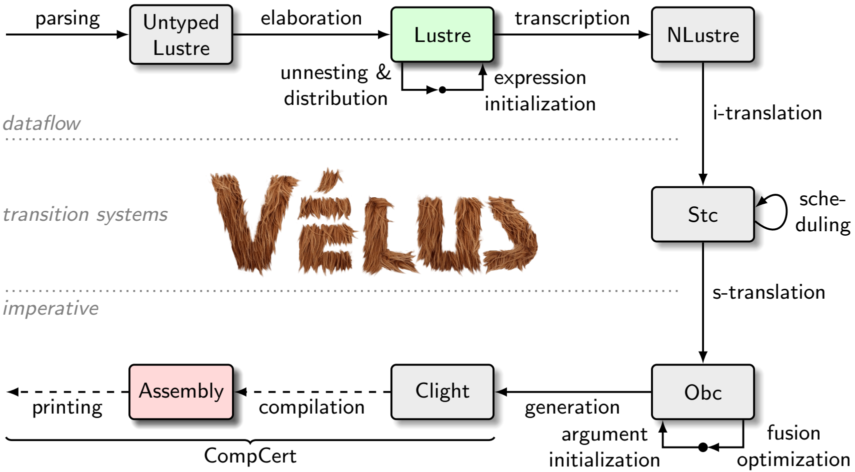
Interactive approach:

- load definitions in the proof assistant (ITP)
- use reasoning techniques to manipulate the goal/hypotheses
- obtain a mathematical proof of the result

Interactive verification in Vélus



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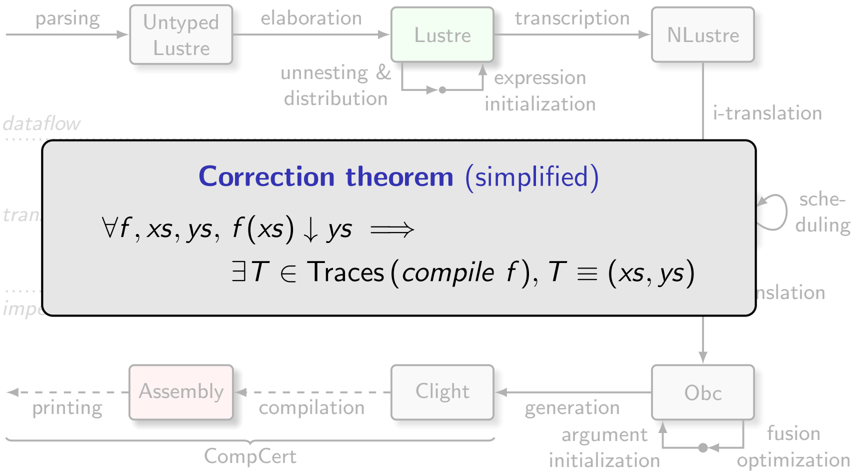

Correction theorem (simplified)

$$\forall f, xs, ys,\ f(xs) \downarrow ys \implies$$
$$\exists T \in \text{Traces}\,(\mathit{compile}\ f),\ T \equiv (xs, ys)$$

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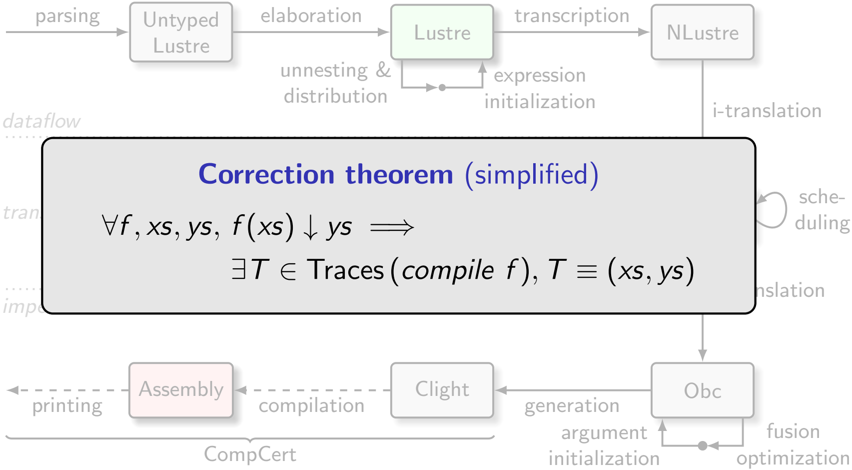

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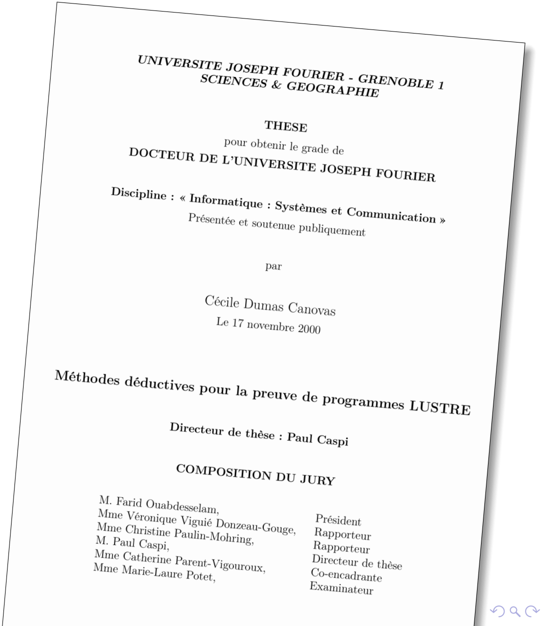

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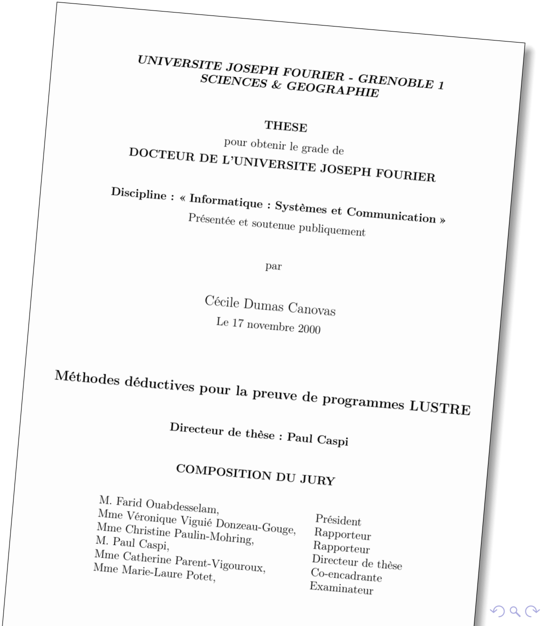

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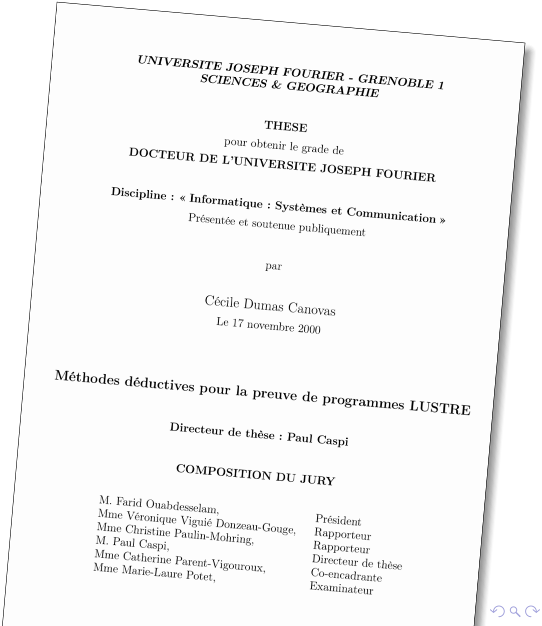

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Three different encodings of streams

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2. Co-induction with infinite proof principle ($\approx$ Vélus)
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   - with a smallest element ϵ
   - stream operations are defined as least fix-points

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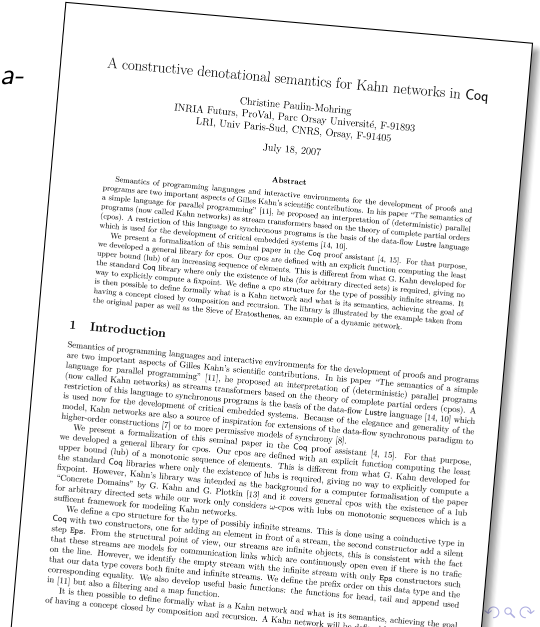
From semantics to CS, 2007



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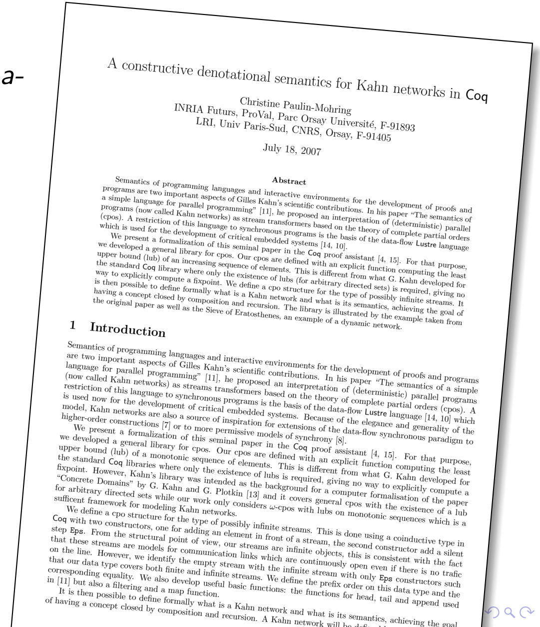

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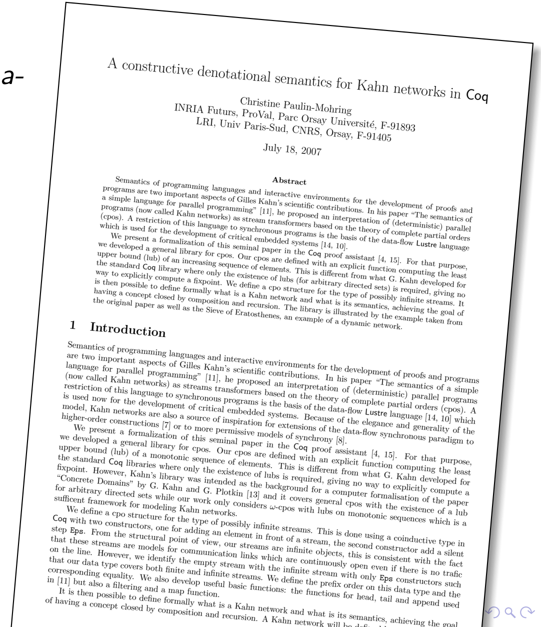

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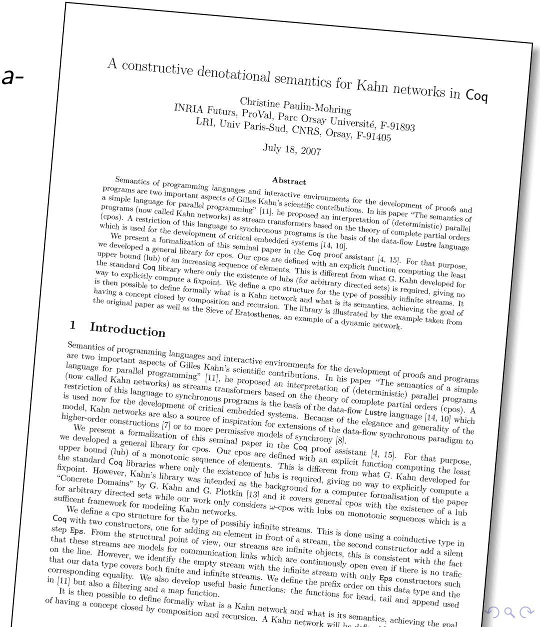

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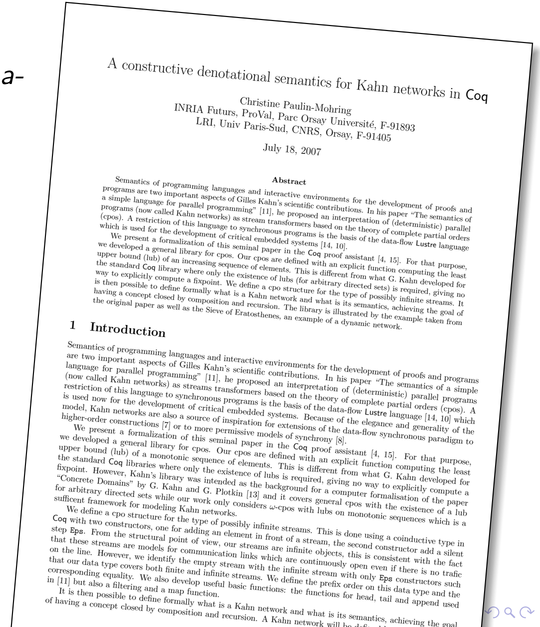
- Example: modeling a (recursive) Kahn network



A constructive denotational semantics for Kahn networks in Coq

Christine Paulin-Mohring
INRIA Futurs, ProVal, Parc Orsay Université, F-91893
LRI, Univ Paris-Sud, CNRS, Orsay, F-91405

July 18, 2007

Abstract

Semantics of programming languages and interactive environments for the development of proofs and programs are two important aspects of Gilles Kahn's scientific contributions. In his paper "The semantics of a simple language for parallel programming" [11], he proposed an interpretation of (deterministic) parallel programs (now called Kahn networks) as stream transformers based on the theory of complete partial orders (cpos). A restriction of this language to synchronous programs is the basis of the data-flow Lustre language which is used for the development of critical embedded systems [14, 10].

We present a formalization of this seminal paper in the Coq proof assistant [4, 15]. For that purpose, we developed a general library for cpos. Our cpos are defined with an explicit function computing the least upper bound (lub) of an increasing sequence of elements. This is different from what G. Kahn developed for the standard Coq library where only the existence of lubs (for arbitrary directed sets) is required, giving no way to explicitly compute a fixpoint. We define a cpo structure for the type of possibly infinite streams. It is then possible to define formally what is a Kahn network and what is its semantics, achieving the goal of having a concept closed by composition and recursion. The library is illustrated by the example taken from the original paper as well as the Sieve of Eratosthenes, an example of a dynamic network.

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We present a formalization of this seminal paper in the Coq proof assistant [4, 15]. For that purpose, we developed a general library for cpos. Our cpos are defined with an explicit function computing the least upper bound (lub) of a monotonic sequence of elements. This is different from what G. Kahn developed for the standard Coq libraries where only the existence of lubs is required, giving no way to explicitly compute a fixpoint. However, Kahn's library was intended as the background for a computer formalisation of the paper "Concrete Domains" by G. Kahn and G. Plotkin [13] and it covers general cpos with the existence of a lub for arbitrary directed sets while our work only considers ω-cpos with lubs on monotonic sequences which is a sufficent framework for modeling Kahn networks.

We define a cpo structure for the type of possibly infinite streams. This is done using a coinductive type in Coq with two constructors, one for adding an element in front of a stream, the second constructor add a silent step Eps. From the structural point of view, our streams are infinite objects, this is consistent with the fact that these streams are models for communication links which are continuously open even if there is no trafic on the line. However, we identify the empty stream with the infinite stream with only Eps constructors such that our data type covers both finite and infinite streams. We define the prefix order on this data type and the corresponding equality. We also develop useful basic functions: the functions for head, tail and append used in [11] but also a filtering and a map function.

It is then possible to define formally what is a Kahn network and what is its semantics, achieving the goal of having a concept closed by composition and recursion. A Kahn network will be

Using previous works

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From semantics to CS, 2007

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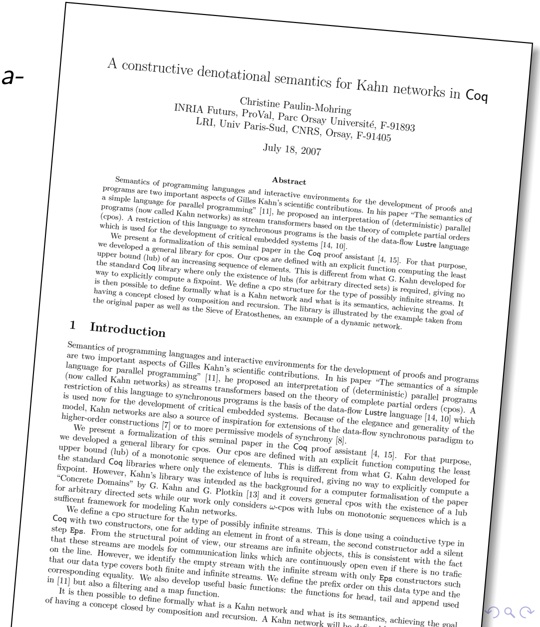

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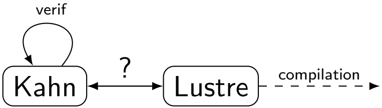

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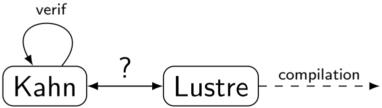

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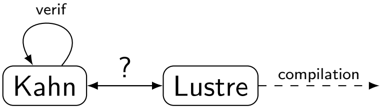

- Is the Kahn semantics suitable for some compilation steps?
- What about the *existence* of a semantics?